# Sigmoidal Function Based Optimal Weighted Vector Directional Filters

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## Abstract

In this paper, we provide a new directional optimization of a recently developed class of weighted vector directional filters (WVDFs) that represent a powerful tool for removing impulsive noise, bit errors and color artifacts in color images. Depending on the weight coefficients, the WVDFs can be designed to perform a wide range of smoothing operations. The proposed optimization process based on sigmoidal approximation of the sign function and utilizing local information allows to adapt the WVDF behavior to statistical properties of both noise and useful signal, saves the memory space and is easy to implement. The proposed sigmoidal function based optimal WVDFs find the application in denoising of standard color images and digitized old color photos, they are able to remove impulsive noise and outliers, and provide excellent signal-detail and color chromaticity preservation.

## Introduction

In color image filtering, a class of vector filters [1-6] respects the inherent correlation that exists between color channels and thus, vector filters avoid a production of color artifacts on which the human visual system is very sensitive. If the noise corruption is characterized by heavy-tailed distribution (e.g. impulsive noise or bit errors), vector filters based on robust order-statistic theory and outputting the sample associated with the minimum distance function are usually preferred. A new class of weighted vector directional filters (WVDFs) outputs the input multichannel sample minimizing the distance function given by the sum of weighted angles to other input samples. Note that each input sample is associated with the nonnegative real weight. A class of WVDFs provides excellent properties such as design flexibility, simple structure, optimal estimates in the sense of color chromaticity preservation, and significant improvement in comparison with standard vector directional filters. Because of a wide range of smoothing operations done by WVDFs, an optimization of the weight vector should be emphasized in the filter design.

The proposed sigmoidal function based optimal WVDFs find the application in denoising of standard color images and digitized old color photos, they are able to remove impulsive noise and outliers, and provide the excellent signal-detail and color chromaticity preservation. The excellent performance of the proposed method will be illustrated in the form of tables and object-lesson images including error signals, row functions and graphical dependencies of used mean absolute error, mean square error and normalized color difference criteria vs. a wide range of impulsive noise probability. The complete analysis of the sigmoidal function based optimization will be provided as well.

## Weighted Vector Directional Filters

Let $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N$ be a set of multichannel vector-valued samples spawned by a filter window of a finite size $N$ and let $\mathbf{x}_{(N+1)/2}$ be a central sample corresponding to the window reference position. Let us consider that $w_1, w_2, ..., w_N$ represent a set of positive real weights, where each weight $w_j$, for $j = 1,2,...,N$, is associated with the input sample $\mathbf{x}_j$. Then, the sum of weighted angular distances [3] associated with the input sample $\mathbf{x}_i$ is given by

$$\beta_i = \sum_{j=1}^{N} w_j \mathrm{A}(\mathbf{x}_i, \mathbf{x}_j) \quad \text{for } i = 1,2,...,N \tag{1}$$

where

$$\mathrm{A}(\mathbf{x}_i, \mathbf{x}_j) = \cos^{-1}\left(\frac{\mathbf{x}_i . \mathbf{x}_j^T}{|\mathbf{x}_i| . |\mathbf{x}_j|}\right) \tag{2}$$

represents the angle [4] between two $m$-dimensional vectors $\mathbf{x}_i = (x_{i1}, x_{i2}, ..., x_{im})$ and $\mathbf{x}_j = (x_{j1}, x_{j2}, ..., x_{jm})$. If the ordering scheme of ordered angular measures

$$\beta_{(1)} \le \beta_{(2)} \le ... \le \beta_{(N)} \tag{3}$$

is implied to the input vector-valued samples $\mathbf{x}_1(\beta_1), \mathbf{x}_2(\beta_2), ..., \mathbf{x}_N(\beta_N)$, it results in the ordered input set

$$\mathbf{x}^{(1)}; \mathbf{x}^{(2)}; ...; \mathbf{x}^{(N)} \tag{4}$$

The output of the WVDF is the sample $\mathbf{x}^{(1)} \in \{\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N\}$ associated with the minimum weighted angular distance $\beta_{(1)} \in \{\beta_1, \beta_2, ..., \beta_N\}$. Thus, the WVDFs are outputting the sample from the input set, so that the local distortion is minimized [3].

If all weight coefficients are set to the same value, all angular distances will have the same importance and the WVDF operation will be equivalent to the well-known basic vector directional filter (BVDF) [6]. If only the center weight is altered whereas other weights remain equal to one, the WVDFs perform the center weighted vector directional filtering (CWVDF) [3].

## Proposed Optimization

A variety of smoothing operations provided by the WVDFs represent the sufficient motivation for the optimization of the weight coefficients. In general, the filter optimization belongs to the most important tasks related to the filter design. The relationship between the pixel under consideration (window center) and each pixel in the filter window should be reflected in the decision for the weight coefficients. In the adaptive design, the weights provide the degree to which the input vector contributes to the output of the filter. In this paper, we determine adaptively the optimal weight vector using directionally generalized sigmoidal optimization of the weighted median (WM) filters [7].

Consider the input set of scalar samples written as $\{x_1(n), x_2(n), ..., x_N(n)\}$ and the original (desired) sample $o(n)$ associated with the time position $n$ determined by the central sample of a running filter window. In the case of sigmoidal approximation, the adaptive optimization algorithm derived from the stack filter design [7] can be simplified to the following expression

$$w_i(n+1) = P\left[w_i(n) + 2\mu(o(n) - y(n))\operatorname{sgn}_S(x_i(n) - y(n))\right] \quad (5)$$

where $w_i(n)$, for $i = 1, 2, ..., N,$ is the filter weight, $y(n)$ is the WM output, $\mu$ is the iteration constant, $\operatorname{sgn}_s(.)$ is a sign function approximated by sigmoidal function

$$\operatorname{sgn}_S(a) = \frac{2}{1+e^{-a}} - 1 \quad (6)$$

and $P(.)$ characterizes a projection operation

$$w_i(n+1) = \begin{cases} w_i(n) & \text{for } w_i(n) \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

If the actual WM output is smaller than the original value, the weights corresponding to the samples that are larger than the actual output are incremented. However, in the case of sigmoidal approximation of the sign function, the convergence to a global optimal solution cannot be guaranteed [7]. In general, the adaptive WM algorithms such as the optimization based on sigmoidal approximation and also linear approximation of the sign function save the memory space, they are easy to implement and provide good results for the time-varying statistics.

Now, let $\{\mathbf{x}_1(n), \mathbf{x}_2(n), ..., \mathbf{x}_N(n)\}$ be the input set of $m$-channel samples and $\mathbf{o}(n)$ the desired (original or noise-free) sample. Let us consider that each input sample $\mathbf{x}_i(n)$ be associated with nonnegative real weight $w_i$, for $i = 1, 2, ..., N.$ Then, we can modify the sigmoidal optimization (5) as its directional generalization for the multichannel case as follows:

$$w_i(n+1) = P\Big[w_i(n) + 2\mu A\big(\mathbf{o}(n), \mathbf{y}(n)\big) S\big(|\mathbf{o}(n)| - |\mathbf{y}(n)|\big). \\ .\operatorname{sgn}_S\Big(A\big(\mathbf{x}_i(n), \mathbf{y}(n)\big) S\big(|\mathbf{x}_i(n)| - |\mathbf{y}(n)|\big)\Big)\Big] \quad (8)$$

where $\mathbf{y}(n)$ is the WVDF output related to the actual weight coefficients $w_1(n), w_2(n), ...., w_N(n)$ and time position $n$. Notation $\mathbf{x}_i(n)$ characterizes the input sample with the $i$-th position in the filter window and $S(\cdot) \in \{-1, 1\}$ is a polarity function.

It is clear that the main difficulty related to the extension of the scalar expression (5) to the multichannel case is the modification of the sign function. The reason is the difficulty of determining the polarity of the distance measure between two multichannel samples. In order to solve this problem, we determine the polarity of the distance measure according to the difference between magnitudes of multichannel samples. In general, applying the used transformation to multichannel samples $\mathbf{a}$ and $\mathbf{b}$ results in the following expression:

$$S(|\mathbf{a}| - |\mathbf{b}|) \begin{cases} +1 & \text{for } |\mathbf{a}| - |\mathbf{b}| > 0 \\ -1 & \text{for } |\mathbf{a}| - |\mathbf{b}| < 0 \end{cases} \quad (9)$$

## Experimental Results

The used color test images Lena, Peppers and Parrots are shown in Figure 1a-c. The efficiency of the methods was evaluated for a wide range of the impulsive noise corruption. As the objective measures [5] we used mean absolute error (MAE), mean square error (MSE) and normalized color difference (NCD). These criteria provide a good mirror of the signal-detail preservation (MAE), the noise attenuation capability (MSE) and the measure of the color distortion (NCD) present in the image, respectively.

The achieved error criteria normalized by the maximum error values related to the WVDF optimization and a various degree of the impulsive noise corruption [5] are shown in Figure 2. Note that the sigmoidal optimization is starting with the initialization of the weight vector as the vector of positive values. It can be seen that the success of the adaptation of the filter weights depends on the iteration constant $\mu$ that should be optimally set to a value greater than 0.1. If $\mu$ is smaller than this sub-optimal value, the sigmoidal WVDF filter provides worse detail preserving characteristics and after some critical point dependent on statistical properties of the training sequence, it will converge to the BVDF operation. Note that the optimization process of the $3\times3$ was started with the same initial weight vector $\mathbf{w}(0) = [1,1,1,1,1,1,1,1,1]$ that corresponds to the BVDF.


*Figure 1. Achieved results. (a) test image Lena, (b) test image Peppers, (c) test image Parrots, (d) 5% impulsive noise, (e) VMF output, (f) BVDF output, (g) DDF output , (h) Proposed method*

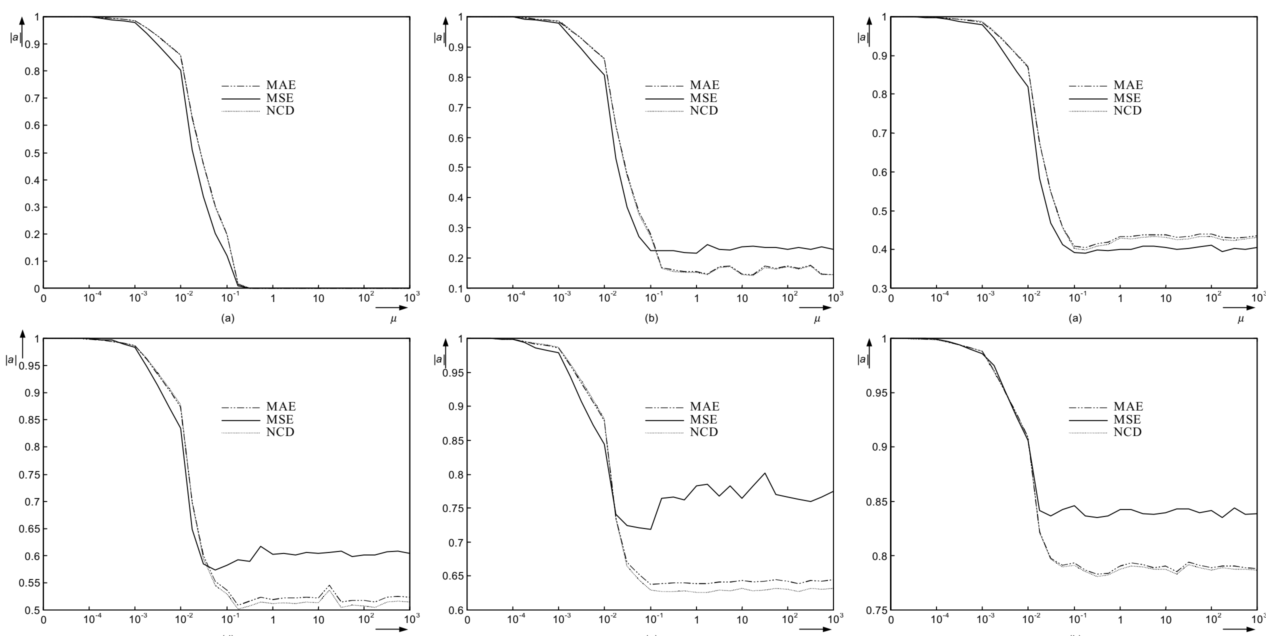


*Figure 2 . WVDF optimization vs. normalized measures in the dependence on the iteration stepsize $\mu$ . Training set was given by the image Lena with no corruption (a) and with impulsive noise: (b) 2% noise, (c) 5% noise, (d) 10% noise, (e) 15% noise, (f) 20% noise*

In order to test the robustness of the method, in the reminder of this paper we used the weight coefficients related to the training set achieved using by the test image Lena corrupted by 10% impulsive noise. We compared the performance of the proposed method with the performance of widely used vector filtering approaches such as vector median filter (VMF) [1], basic vector median filter (BVDF) [6] and directional distance filter (DDF) [2].

It can be seen (Figures 3-7 and Tables 1-6) that the proposed method can achieve interesting improvement of the filter performance in comparison with the relevant filtering techniques such as VMF, BVDF, DDF and non-optimized WVDF with the weight vector $\mathbf{w}=[1,2,1,4,5,4,1,2,1]$. In addition, the proposed method is able to achieve the excellent balance between the noise attenuation characteristics and signal-detail preservation characteristics.

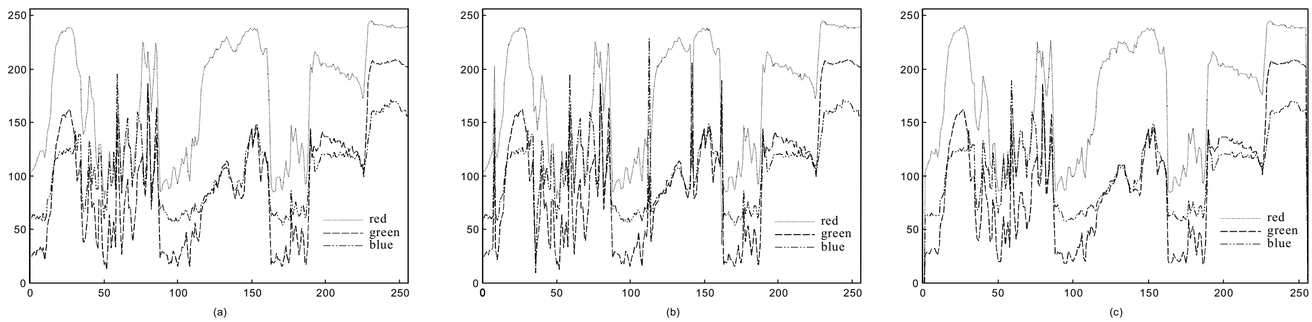


*Figure 3. Row functions (180th row) related to the image Lena: (a) original image, (b) 2% noise, (c) output of the proposed method*

**Table 1 Achieved results using the image Peppers**

| Method | MAE | MSE | NCD |
|---|---|---|---|
| 2% noise | 1.582 | 197.6 | 0.0178 |
| VMF | 3.011 | 39.7 | 0.0428 |
| BVDF | 3.585 | 55.9 | 0.0418 |
| DDF | 3.027 | 40.7 | 0.0411 |
| WVDF | 1.992 | 28.7 | 0.0251 |
| proposed method | 1.708 | 24.6 | 0.0205 |

**Table 2 Achieved results using the image Parrots**

| Method | MAE | MSE | NCD |
|---|---|---|---|
| 2% noise | 1.578 | 186.2 | 0.0177 |
| VMF | 2.493 | 58.0 | 0.0123 |
| BVDF | 3.289 | 101.8 | 0.0106 |
| DDF | 2.482 | 60.2 | 0.0108 |
| WVDF | 1.879 | 53.3 | 0.0058 |
| proposed method | 1.433 | 45.2 | 0.0043 |

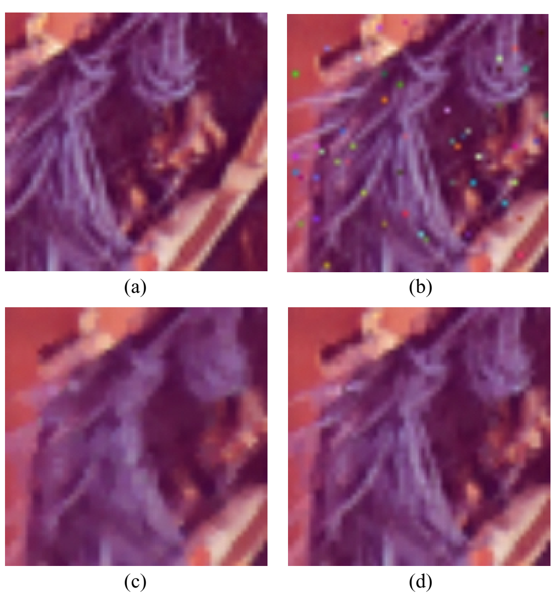


*Figure 4. Zoomed results obtained using the test image Lena. (a) original image, (b) noisy image (2% impulsive noise), (c) VMF output, (d) output of the proposed method*

**Table 3 Achieved results using the image Peppers**

| Method | MAE | MSE | NCD |
|---|---|---|---|
| 5% noise | 3.988 | 486.1 | 0.0441 |
| VMF | 3.169 | 43.9 | 0.0452 |
| BVDF | 3.740 | 60.7 | 0.0438 |
| DDF | 3.182 | 44.6 | 0.0431 |
| WVDF | 2.197 | 38.1 | 0.0275 |
| proposed method | 1.876 | 33.9 | 0.0227 |

**Table 4 Achieved results using the image Parrots**

| Method | MAE | MSE | NCD |
|---|---|---|---|
| 5% noise | 3.805 | 443.6 | 0.0432 |
| VMF | 2.669 | 64.2 | 0.0132 |
| BVDF | 3.460 | 109.0 | 0.0116 |
| DDF | 2.645 | 65.3 | 0.0117 |
| WVDF | 2.061 | 62.2 | 0.0068 |
| proposed method | 1.574 | 50.4 | 0.0052 |

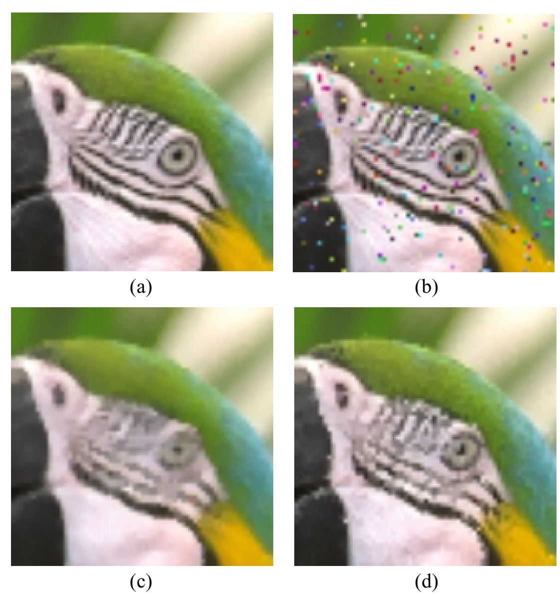


*Figure 5. Zoomed results obtained using the test image Parrots. (a) original image, (b) noisy image (5% impulsive noise), (c) VMF output, (d) output of the proposed method*

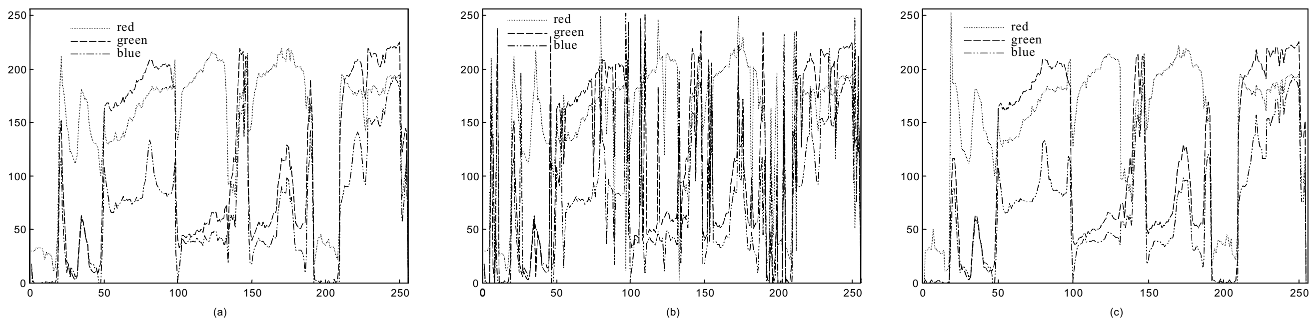


*Figure 6. Row functions (100th row) related to the image Peppers: (a) original image, (b) 15% noise, (c) output of the proposed method*

**Table 5 Achieved results using the image Peppers**

| Method | MAE | MSE | NCD |
|---|---|---|---|
| 10% noise | 7.677 | 943.3 | 0.0870 |
| VMF | 3.503 | 55.0 | 0.0494 |
| BVDF | 4.151 | 82.7 | 0.0484 |
| DDF | 3.512 | 56.6 | 0.0475 |
| WVDF | 2.659 | 65.9 | 0.0324 |
| proposed method | 2.330 | 67.3 | 0.0274 |

**Table 6 Achieved results using the image Parrots**

| Method | MAE | MSE | NCD |
|---|---|---|---|
| 10% noise | 7.526 | 882.0 | 0.0857 |
| VMF | 2.890 | 69.6 | 0.0142 |
| BVDF | 3.630 | 113.5 | 0.0127 |
| DDF | 2.839 | 69.7 | 0.0128 |
| WVDF | 2.362 | 77.2 | 0.0083 |
| proposed method | 1.831 | 62.6 | 0.0065 |

(a) (b) (c)

*Figure 7. Estimation errors emphasized by factor 2.5 related to the test image Lena degraded by 2% impulsive noise: (a) VMF, (b) BVDF, (c) proposed method*

## Conclusion

We presented the directional sigmoidal optimization of the weighted vector directional filters (WVDFs). The successful adaptation of the WVDFs to varying image statistics was proven by presented results. From these results it can be seen that the proposed method clearly outperforming the standard vector filters such as VMF, BVDF and DDF. The proposed optimization is fast, saves the memory space and is easy to implement. After the optimization, the proposed WVDFs are sufficiently robust and useful for practical image applications.

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## Biography

**Dr. Rastislav Lukac** received the M.Sc. (Ing.) degree with honors in 1998 and the Ph.D. degree in 2001, both at the Technical University of Kosice, the Slovak Republic. Recently, his research interests include nonlinear digital filters, impulse detection, color image processing, image sequence processing and the use of Boolean functions and permutation theory in filter design. He is an author or a co-author of over 90 scientific papers published in journals and conference proceedings. Dr. Lukac is a member of the IEEE Signal Processing Society.

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